$\therefore$ PQ and GO bisect one another at M. From the triangles MQO and GPM, $GM=OM$, $PM=QM$, $\angle OMQ=\angle PMG$. $\therefore$ $OQ=PG=FP$, and OQ is also parallel to FP. $\therefore$ $OF=PQ$.

$\therefore$ The radius of EFD=the diameter of ABC. Perpendicular to FG at the point P draw $PO'=QD$. Then since $FP=PG=OQ$, $O'F=O'G=OD$.

$\therefore$ $OF=O'F=OD=O'D=O'G$.

$\therefore$ radius of FGD=radius EFD=diameter ABC.

Similarly radius EGD and radius EGF=radius (each) EFD.

98. Proposed by EDWARD R. ROBBINS, Master in Mathematics and Physics, Lawrenceville School, Lawrenceville, N. J.

Construct a circle which shall pass through two given points and touch a given circle, (1) when the distance between the points is less than the diameter of the circle, and (2) when it is greater.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

The construction for both cases is the same.

I Case. Let A, B be the given points, $CMDK$ the given circle. Through A, B draw the circle $ABEF$ intersecting the given circle in E, F. Draw AB, EF intersecting at G. Draw the tangents GK, GM. Draw QQ_1 perpendicular to AB at its mid-point. Through R, the center of $CMDK$, draw RM, RK intersecting QQ_1 in O_1, O. Then O, O_1 are the centers of two circles satisfying the conditions, and ABK, ABM are the circles.

II Case. Let C, D be the given points, $AHBL$ the given circle. Through C, D describe the circle $CDEF$ intersecting the given circle in E, F. Draw CD, EF intersecting at G. Draw the tangents GH, GL. Draw RR_1 perpendicular to CD at its mid-point. Through Q, the center of $AHBL$, draw QL, QH intersecting RR_1 in P, P_1. Then P, P_1 are the centers of two circles satisfying the conditions, and CDL, CDH are the circles.

In the above both points are without the given circle. This problem is thoroughly discussed on page 271, No. 8, Vol. I., THE AMERICAN MATHEMATICAL MONTHLY.

II. Solution by FREDERIC R. HONEY, Ph. B., New Haven, Conn.

The following description applies when the distance between the points is less, and when greater than the diameter of the circle.

Let a and b be the given points and A the circumference of the given circle.

Through a and b pass a circle the circumference of which intersects A at c and d. Draw ba and dc and produce these lines until they meet at e. Draw ef tangent to A. Through the point of tangency f and the given points a and b pass the required circle C. Since two tangents may be drawn there are, in each case, two solutions.

Analysis of the construction: $eb\times ea=ed\times ec=(ef)^2$.

[NOTE. For a demonstration of this same proposition with a diagram, see Vol. I., page 271. Professors Zerr and Honey each furnished neat diagrams with these demonstrations, but we believe the demonstrations sufficiently clear without them. ED. F.]